Localization

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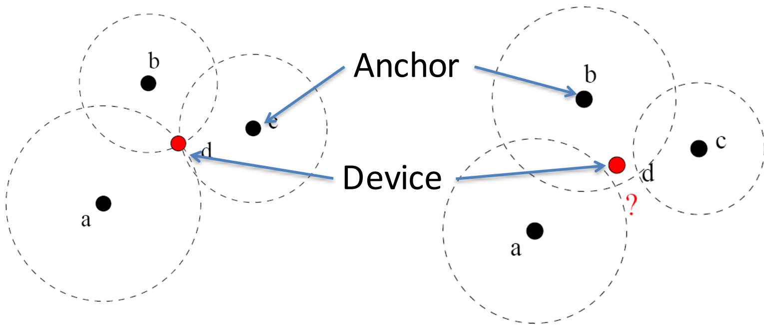
1

Localization Problem

- How to localize (determine coordinates of a device in 2D or 3D space) using radio signals.
- Important problem to solve for supporting location driven application and protocols.
  - Example applications: navigation, location-based services.
- Two basic types of approaches
  - Ranging based
  - Fingerprint based

2

Ranging-based: Trilateration



- Convert received signal strength (RSS) or signal timing to a distance estimate with respect to anchor nodes with known locations.
- Problem: distance estimates may be erroneous and the circles may not intersect at a single point.

3

Approach

- How to estimate location when the circles do not intersect?
- Idea: localize at a point that presents the minimum error to the circles by some reasonable error measure.
- k anchors at positions (x_i, y_i)
- Assume node to be localized has actual location at (x_0, y_0)
- Distance estimate between node 0 and anchor *i* is r_i
- Error:

$$f_i = r_i - \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2}$$

[source: Jie Gao's lecture slides]

1/29/09 Jie Gao, CSE595-spring09 4

Linearization and Min Mean Square Estimate

- Ideally, we would like the error to be 0

$$f_i = r_i - \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2} = 0$$

- Re-arrange:

$$({x_0}^2 + {y_0}^2) + x_0(-2x_i) + y_0(-2y_i) - {r_i}^2 = -x_i^2 - y_i^2$$

- Subtract the last equation from the previous ones to get rid of quadratic terms.

$$2x_0(x_k - x_i) + 2y_0(y_k - y_i) = {r_i}^2 - {r_k}^2 - x_i^2 - y_i^2 + x_k^2 + y_k^2$$

- Note that this is linear.

[source: Jie Gao's lecture slides]

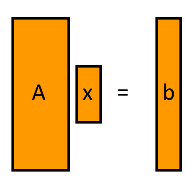
Linearization and Min Mean Square Estimate

- In general, we have an over-constrained linear system

$$Ax = b$$

$$b = \begin{bmatrix} r_1^2 - r_k^2 - x_1^2 - y_1^2 + x_k^2 + y_k^2 \\ r_2^2 - r_k^2 - x_2^2 - y_2^2 + x_k^2 + y_k^2 \\ \vdots \\ r_{k-1}^2 - r_k^2 - x_{k-1}^2 - y_{k-1}^2 + x_k^2 + y_k^2 \end{bmatrix} \quad A = \begin{bmatrix} 2(x_k - x_1) & 2(y_k - y_1) \\ 2(x_k - x_2) & 2(y_k - y_2) \\ \vdots & \vdots \\ 2(x_k - x_{k-1}) & 2(y_k - y_{k-1}) \end{bmatrix}$$

$$x = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$



[source: Jie Gao's lecture slides]

Solve using the Least Square Equation

The linearized equations in matrix form become

$$Ax = b$$

Now we can use the least squares equation to compute the location estimate.

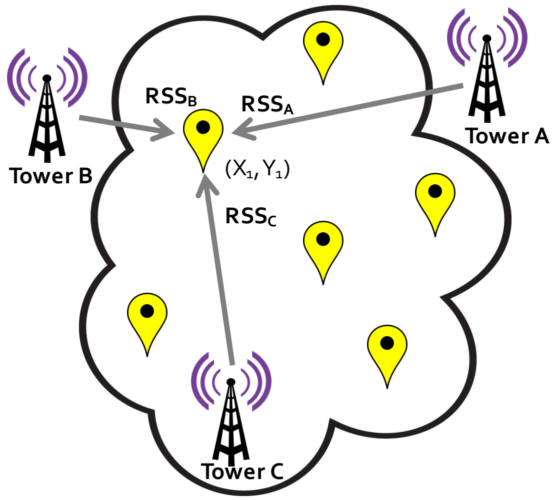
$$x = (A^T A)^{-1} A^T b$$

[source: Jie Gao's lecture slides]

Fingerprinting-based Approach

- In many real-world applications using cellular and WiFi signals, ranging-based method does not provide good accuracy.
- This motivates thinking about alternatives.
- Fingerprinting-based approach does not depend on distance estimates that could be erroneous, but depends on prior radio signal survey on the region of interest (called fingerprinting).

Fingerprinting



Fingerprint Database

Location	Feature Vector
X_1, Y_1	$< RSS_A\ RSS_B\ RSS_C >_1$

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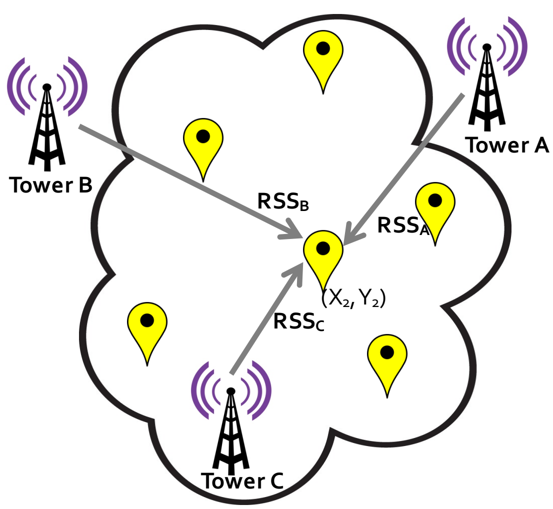
Fingerprinting



Fingerprint Database

Location	Feature Vector
X_1, Y_1	$< RSS_A\ RSSI_B\ RSS_C >_1$
X_2, Y_2	$< RSS_A\ RSS_B\ RSS_C >_2$

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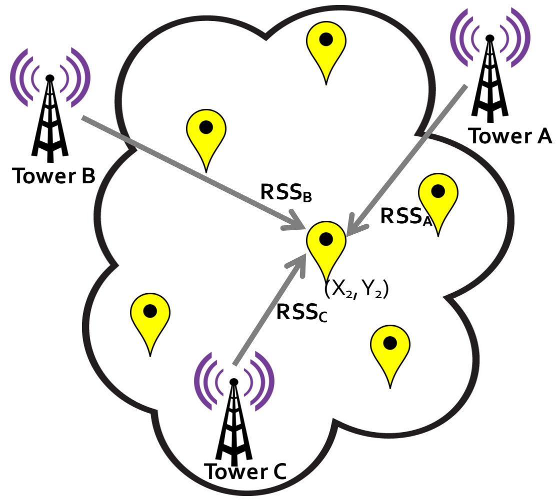
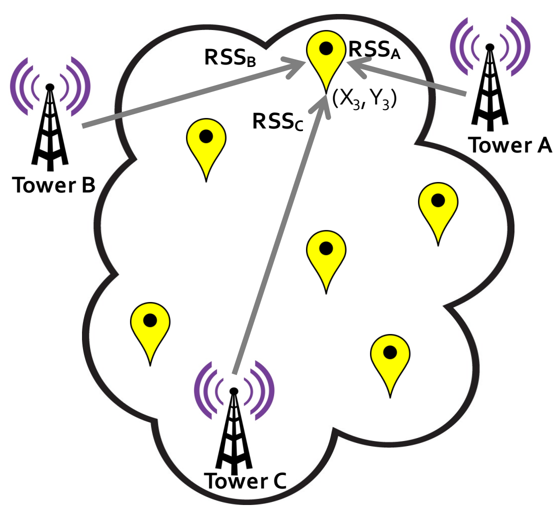
Fingerprinting



Fingerprint Database

Location	Feature Vector
X_1, Y_1	< RSS_A $RSSI_B$ RSS_C>$_1$
X_2, Y_2	< RSS_A RSS_B RSS_C>$_2$

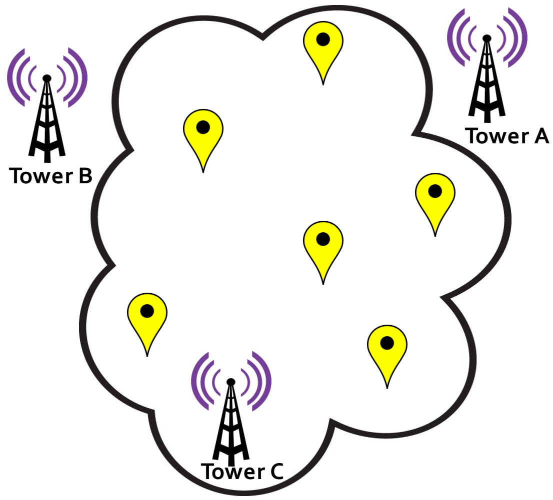
Fingerprinting



Fingerprint Database

Location	Feature Vector
X_1, Y_1	< RSS_A RSS_B RSS_C>$_1$
X_2, Y_2	< RSS_A RSS_B RSS_C>$_2$
X_3, Y_3	< RSS_A RSS_B RSS_C>$_3$

Fingerprinting



Fingerprint Database

Location	Feature Vector
X_1, Y_1	$< RSS_A\ RSS_B\ RSS_C >_1$
X_2, Y_2	$< RSS_A\ RSS_B\ RSS_C >_2$
X_3, Y_3	$< RSS_A\ RSS_B\ RSS_C >_3$
X_4, Y_4	$< RSS_A\ RSS_B\ RSS_C >_4$
X_5, Y_5	$< RSS_A\ RSS_B\ RSS_C >_5$
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X_N, Y_N	$< RSS_A\ RSS_B\ RSS_C >_N$

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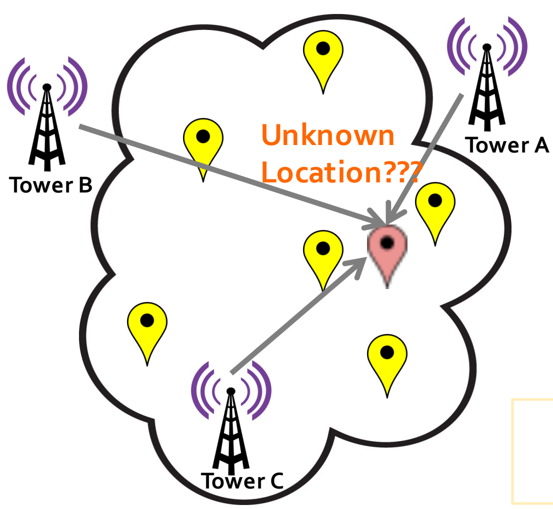
13

Fingerprinting



Fingerprint Database

Location	Feature Vector
X_1, Y_1	$< RSS_A\ RSS_B\ RSS_C >_1$
X_2, Y_2	$< RSS_A\ RSS_B\ RSS_C >_2$
X_3, Y_3	$< RSS_A\ RSS_B\ RSS_C >_3$
X_4, Y_4	$< RSS_A\ RSS_B\ RSS_C >_4$
X_5, Y_5	$< RSS_A\ RSS_B\ RSS_C >_5$
▪▪▪ ▪▪▪ ▪▪▪	
X_N, Y_N	$< RSS_A\ RSS_B\ RSS_C >_N$

Measured RSS vector
$< RSS_A\ RSS_B\ RSS_C >$.
What is the location??

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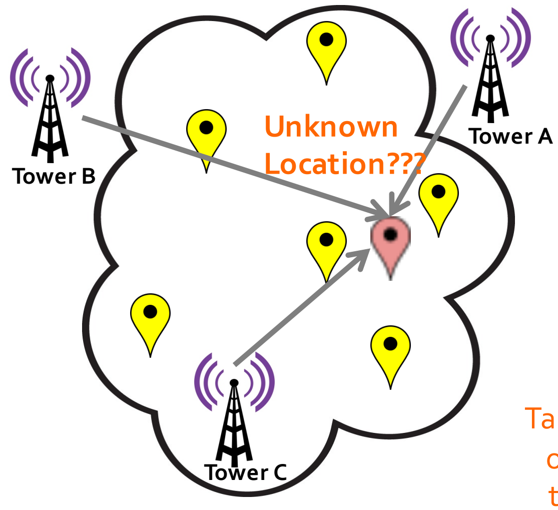
14

Fingerprinting



Fingerprint Database

Location	Feature Vector
X_1, Y_1	$< RSS_A\ RSS_B\ RSS_C >_1$
X_2, Y_2	$< RSS_A\ RSS_B\ RSS_C >_2$
X_3, Y_3	$< RSS_A\ RSS_B\ RSS_C >_3$
X_4, Y_4	$< RSS_A\ RSS_B\ RSS_C >_4$
X_5, Y_5	$< RSS_A\ RSS_B\ RSS_C >_5$
... ...	
X_N, Y_N	$< RSS_A\ RSS_B\ RSS_C >_N$

K-Nearest Neighbor: Take an average of locations of K nearest neighbors in terms of feature vectors.

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15

Fingerprinting-based Approach

- K-Nearest Neighbor is a basic technique. More sophisticated, statistics-based techniques are possible and provide better results.
- Fingerprinting-based approaches could be of high cost as building the fingerprint itself could be expensive in terms of labor, given that a denser fingerprinting can provide better accuracy.

16